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Numerical Method (Assignment)

① $\frac{dy}{dx} = -y^2$

$x_0 = 1, \quad y_0 = 1$

$y(1.2) = ?$

$h = 0.1$

$x_0 = 1$

$x_1 = x_0 + xh = 1 + 0.1$

$x_1 = 1 + 2 \times 0.1$

$= 1 + 0.2 = 1.2$

① Euler method

$y_{n+1} = y_n + hf(x_n, y_n)$

put $n = 0$

$y_1 = y_0 + hf(x_0, y_0)$

$= 1 + 0.1(-1^2)$

$= 1 - 0.1 = 0.9$

$y_2 = y_1 + hf(x_1, y_1)$

$= 0.9 + 0.1(0.9)^2$

$= 0.819$

Hence, $\boxed{y(1.2) = 0.819}$

(ii) Backward Euler method

$$y_1 = y_0 + hf(x_0, y_0) = 1 + 0.1(-1)^2$$
$$= 0.9$$

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1)\right]$$

$$= 1 + \frac{0.1}{2}\left[(-1)^2 + (-0.9)^2\right] = 1 + 0.05\left[-1 - 0.8\right]$$

$$= 1 - 0.0905 = 0.9095$$

$$y_2 = y_1 + hf(x_1, y_1) = 0.9095 + 0.1\left[\left(-(9095)^2\right]\right.$$
$$= 0.995 - 0.0827 = 0.9123$$

$$y_2^2 = y_1 + \frac{x}{2}\left[f(x_1, y_1) + f(x_2, y_2)\right]$$

$$= 0.9095 + \frac{0.1}{2}(0.9095)^2 + -(.9123)^2\big]$$

$$= 0.9095 + \frac{0.1}{2}(-1.6504)$$

$$= 0.82653 \quad \underline{\text{Ans}}$$

(iii) midpoint method

$$x_i + \frac{1}{2} = x_i + \frac{h}{2}$$

$$y_i + \frac{1}{2} = y_i + hf\ (x_i\ y_i)$$

$$y_i + 1 = y_i + hf\left(x_i + \frac{1}{2}, y_i + \frac{1}{2}\right)$$

$$y_1 = y_0 + 0.1 f\left(x_{1/2}, y_{1/2}\right)$$

$$= 1 + 0.1\left(-0.9^2\right)$$
$$= 1.081$$
$$= 0.919$$

Now, $x_{3/2} = x_i + \frac{h}{2} = 1.1 + \frac{0.1}{2} = 1.15$

$$y_2 = y_1 + x(x_{3/2}, y_{3/2})$$

$$y_2 = 0.919 + 0.1(-(0.8345)^2)$$

$$\boxed{y_2 = 0.8495}$$

3 $\frac{\delta y}{\delta x} = x + y$

$x_0 = 1$
$y_0 = 1$
$h = 0.1$

$x_1 = x_0 + 4h = 1 + 0.1 = 1.1$
$x_2 = x_0 + 0.2 = 1 + 0.2 = 1.2$

Now

$$y_1 = y_0 + hf(x_0, y_0) = 0 + 0.1(x_0, y_0)$$

$$= 0 + 0.1(1 + 0) = 0.1$$

$$y_1' = y_0 + \frac{x}{2}(f(x_0, y_0) + f(x_1, y_1))$$

$$= 0 + \frac{0.1}{2}[(x_0, y_0) + (x_1, y_1)]$$

$$= 0.05 \; [1 + 0 + 1.1 + 0.1) = 0.1$$

$$y_2 = y_0 + hf(x_1, y_1)$$

$$= y_1 + h(x_1, y_1) = 0.1 + 0.1(1.1 + 0.11)$$

$$= 0.231$$

$$y_2' = y_1 + \frac{h}{2}\left[f(x_1 + y_1) + f(x_2 + y_2)\right]$$

$$= 0.11 + \frac{0.1}{2}\left[(x_1 + y_1) + (x_2 + y_2)\right]$$

$$= 0.11 + \frac{0.1}{2}\left[(1.1 + 0.11) + (1.2 + 0.23)\right]$$

$$= 0.11 + \frac{0.1}{2} \times 2.641$$

$$\boxed{y_2^1 = 0.24205}$$

(4) $$\frac{\delta y}{\delta x} = x^3 + 3y$$

$x_0 = 0$

$y_0 = 1$

$h = 0.2$

$y(0.2) = ?$

$y(0.4) = ?$

using Euler method

$4_1 = x_0 + h = 0 + 0.2 = 0.2$

$x_2 = x_0 + 2h = 0 + 2 \times 0.2 = 0.4$

$y_1 = y_0 + h f(x_0, y_0) = y_0 + h(x_0^3 + 3y_0)$

$= 1 + 0.2(0^3 + 3 \times 1) = 1 + 0.2 \times 3$

$\boxed{y_1 = 1 + 0.6 = 1.6}$

$$y_2 = y_1 + hf(x_1, y_1)$$
$$= y_1 + h(x_1^3 + 3y_1)$$

$$y_2 = 1.6 + 0.2\,(0.2^3 + 3 \times 1.6)$$

$$\boxed{y_2 = 2.5616}$$ Ans.

(5) $h = 1, \quad x = 3$
$i = 1, 2$
As i lies b/w 1 & $(x+1)$.
$M_0 = 0, \quad M_3 = 0$

equation is
$$M_{i-1} + 4M_i + M_{i+1} = \frac{6}{x^2}(y_{i-1} - 2y_i + y_{i+1})$$

put $i = 1$
$$M_0 + 4M_1 + M_2 = \frac{6}{h^2}(y_0 - 2y_1 + y_2)$$

$$4M_1 + M_2 = 6(3 - 2 \times 10 + 29)$$

$$4M_1 + M_2 = 6(3 - 20 + 29$$

$$4M_1 + M_2 = 72 \quad \text{— (1)}$$

put $i = 2$
$$M_1 + 4M_2 + M_3 = \frac{6}{x^2}(y_1 - 2y_2 + y_3)$$

$$M_1 + 4M_2 = 6(10 - 2 \times 29 + 65)$$

$$M_1 + 4M_2 = 6(10 - 58 + 65)$$

$$M_1 + 4M_2 = 102 \quad \text{— (11)}$$

from I & II

$I \times 4 - II$

$$16 M_1 + 4\cancel{M_2} - M_1 - 4\cancel{M_2} = 288 - 102$$
$$15 M_1 = 186$$
$$\boxed{M_1 = 12.4}$$

from II

$$12.4 + 4M_2 = 102$$
$$4M_2 = 89.6$$
$$M_2 = 22.4$$

we know that
So, we will know from equation for each internal

$$f(u) = \frac{(x_{i+1} - u)^3}{6x} M_i + \frac{(x - u_i)^3 M_{i+1}}{6h}$$
$$\frac{x_{i+1} - x}{h}\left(y_i - \frac{h^2}{6} M_1\right) + \frac{x - x_i}{h}\left(y_{i+1} - \frac{h^2}{6} M_{i+1}\right)$$

Put $i = 0$
$$x_i < x < x_1$$
that means
$$1 < x < 2$$
$$f(x) = \frac{(x_1 - x)^3}{6} M_0 - \frac{(x - x_0)^3}{6} M_1 + \frac{x_1 - x}{1}\left(y_0 - \frac{h^2}{6} M_0\right)$$
$$+ \frac{x_1 - x_0}{h}\left(y_1 + \frac{h^2}{6} M_1\right)$$
$$= \frac{(2-4)^3}{6} \times 0 + \frac{(4-1)^3}{6} 12.4 + 2 - 4\left(3 - \frac{1}{6} \times 0\right)$$
$$+ \frac{4-1}{1}\left(10 - \frac{1}{6} \times 12.4\right)$$

$= 2.06 (x-1)^3 + (2-4)^3 + (4-1)(7.933)$

$= 2.06 (x^3 - 1 - 3 4)(x-1)]$

$= 2.064^3 - 2.06 - 6.184^2 + 6.18x + 6 - 3x + 7.93 - 7.933$

$= 2.06x^3 - 6.184^2 + 11.114 - 9.993$

Now put $i = 2$

$x_2 < x < x_3$

$3 < x < 4$

By putting $i = 2$

$$f(x) = \frac{(x_3 - x)^2}{6h} M_2 + \frac{(4 - x_2)^3}{6h} M_3$$

$$+ \frac{x_3 - 4}{h} \left(y_2 - \frac{h^2}{6} M_3\right)$$

$$+ \frac{x - x_2}{h} \left(y_3 - \frac{h^2}{6} M_3\right)$$

$= (y-x)^3\, 3.733 + (3.x)(25.266) + (x-3)(6.5)$

$= [64 - x^3 - 124(4-x)]\, 3.7333 + (3-x)(25.266) + (x-3)\, 6.5$

$= (64 - x^3 - 48x + 124^2)\, 3.7333 + (3-x)(25.266) + (x.3)(6.5)$,

$f(x) = 238.912 - 3.733x^3 - 179.184x + 44.796x^2 + 75.798 - 25.266x + 65x - 195$

$= -3.733x^2 + 44.796x^2 - 139.45x + 11.971$

Hence, which spline valid in $(3,4]$ is

$$\boxed{f(4) = -3.733x^2 + 44.796x^2 - 139.45x + 119.7}$$

7) from the given data

Case I :- $0 < 4 < 1$

$$P_1(4) = \frac{4_1 - 4_1}{x_0 - x_1} f(4_0) + \frac{x - x_0}{x_1 - x_0} f(x_1)$$

$$= \frac{x-1}{0-1} \times 1 + \frac{4-0}{1-0} \times 2$$

$$P_1(4) = (1-4) + 24 = 1 - x + 24$$

$$P_1(4) = 4 + 1.$$

Case II :- $1 < x < 2$

$$P_2(4) = \frac{4-4_1}{x_0 - x_1} f(4_0) + \frac{4-4_0}{x_1 - x_0} f(4_1).$$

$$= \frac{4-2}{1-2} \times 2 + \frac{x-1}{2-1} \times 5$$

$$= (2-x)2 + 5(x-1)$$

$$= 4 - 24 + 54 - 5$$

$$P_2(4) = 3x - 1.$$

Case - III. :- $2 < x < 3$

$$P_3(x) = \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(4_1)$$

$$P_3(x) = \frac{x-3}{2-3} \times 5 + \frac{x-2}{3-2} \times 10$$

$= (3-x)5 + (x-2)10$

$= 15 - 5x + 10x - 20 \quad = 5x - 5$

Hence,

$$p(x) = \begin{cases} x+1 & 0 \le x \le 1 \\ 3x-1 & 0 \le x \le 2 \\ 5x-5 & 2 \le x \le 3 \end{cases}$$

Now, interpointing at

(I) $x = 0.5$

$\Rightarrow p(x) = 0.5 + 1 = \boxed{1.5}$

(II) $x = 1.5$

$\Rightarrow p(x) = 3 \times 1.5 - 1 = \boxed{3.5}$

(III) $x = 2.5$

$p(x) = 5 \times 2.5 - 5 = \boxed{7.5}$